Efficient Modal Analysis of Waveguide Filters Including the Orthogonal Mode Coupling Elements by an MM/FE Method

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Abstract— **An efficient hybrid mode-matching finite element (MM/FE) method is applied for the rigorous analysis of waveguide filters composed of homogeneous standard waveguide cavities together with waveguide coupling sections of nearly arbitrary cross-section. To demonstrate the efficiency of the method, a simple two-pole circular waveguide dual-mode filter is analyzed where the orthogonal modes are coupled by obliquely positioned rectangular post elements with rounded edges and the coupling to the rectangular port waveguides is provided by rectangular irises with rounded corners. Moreover, a four-pole filter is shown where the dual-mode coupling is achieved by asymmetrically located irises. The theory is verified by excellent agreement with measurements.**

I. Introduction

THE AVAILABILITY of reliable and efficient CAD tools for waveguide components is of high importance for many applications, such as for space communication purposes where accuracy, compactness, and development time are very often the most critical component design factors [1]. In order to achieve the required accuracy, the consequent utilization of field-theory-based models is indispensable. For components composed of rectangular and circular waveguide structures, efficient mode-matching (MM) key-building block models have already been developed [2], [3]. For the analysis of more complicated structures, pure numerical methods that require a rather high numerical effort, such as the three-dimensional finite element method (FEM), are typically used [4].

Many complicated waveguide components, however, may be considered to be composed of homogeneous sections where most parts are standard geometries and merely a few require the treatment with computationally more intensive methods. This aspect allows the development of flexible but efficient CAD tools. More recently, a combined mode matching and finite difference method has been proposed [5]. In our paper, a combined mode matching finite element (MM/FE) method is applied that allows the flexible and reliable modeling of even very small obstacles, such as coupling elements in dual-mode filters, due to the typical triangular FEM mesh generator.

The two-dimensional eigenvalue problem of ridged circular waveguide coupling sections has been solved recently by a FE method [6], [7]. Moreover, a FE scattering matrix method has been already applied for modeling arbitrarily shaped irises in circular waveguides [7]. However, complete three-dimensional components, such as circular waveguide dual-mode filters, have not yet been investigated by such methods so far. The rigorous multi-mode scattering matrix of dual-mode filters obtained by the MM/FE method, together with the already available rigorous T-junction and iris key-building blocks, allows the overall CAD of manifold multiplexers, taking into account the significant higher-order mode interaction effects of such structures.

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II. Theory

The transverse electric $\vec{E}_t$ and magnetic fields $\vec{H}_t$ in each homogeneous waveguide section are represented by scalar potentials Ψ^H and Ψ^E in the following way:

$$\begin{aligned}\vec{E}_t &= \sum_{m=1}^{\infty}\sqrt{\frac{\omega\mu}{\beta_m^H}}\vec{u}_z \times \nabla_t\Psi_m^H(a_m^H e^{-j\beta_m^H z} + b_m^H e^{j\beta_m^H z}) \\ &\quad - \sum_{m=1}^{\infty}\sqrt{\frac{\beta_m^E}{\omega\epsilon}}\nabla_t\Psi_m^E(a_m^E e^{-j\beta_m^E z} + b_m^E e^{j\beta_m^E z}), \\ \vec{H}_t &= -\sum_{m=1}^{\infty}\sqrt{\frac{\beta_m^H}{\omega\mu}}\nabla_t\Psi_m^H(a_m^H e^{-j\beta_m^H z} - b_m^H e^{j\beta_m^H z}) \\ &\quad - \sum_{m=1}^{\infty}\sqrt{\frac{\omega\epsilon}{\beta_m^E}}\vec{u}_z \times \nabla_t\Psi_m^E(a_m^E e^{-j\beta_m^E z} - b_m^E e^{j\beta_m^E z})\end{aligned} \tag{1}$$

where H and E denote the TE- and TM-modes, respectively, a and b the normalized amplitudes of the waves propagating in $+z$ or $-z$ direction, $\vec{u}_z$ is the unit vector in $+z$ direction, and ∇_t is the transverse part of the nabla operator. The scalar potentials $\Psi^{H,E}$ are solutions of the transverse homogeneous Helmholtz equation, Ψ^H and Ψ^E satisfy Dirichlet and Neumann boundary conditions on perfectly conducting electric Γ_E, and magnetic walls Γ_M show the usual orthonormal properties.

The initial mesh for the two-dimensional FEM solution of the Helmholtz equation for the sections with nearly arbitrary geometry is generated by the Delaunay triangulation [8]; the mesh can be locally refined and optionally smoothed. The potentials Ψ are approximated by their nodal values Ψ_K and first-order Lagrangian interpolation polynomials $N_K(x, y)$ [9]

by

$$\Psi(x,y) \approx \sum_K \Psi_K N_K(x,y). \tag{2}$$

Inserting (2) in the appropriate functional for the Helmholtz equation

$$F(\Psi) = \int\int_\Omega [(\nabla_t \Psi)^2 - k_c^2 \Psi^2] d\Omega, \tag{3}$$

where $k_c^2 = k^2 - \beta^2$, k is the free space wave number, and extremizing (3) with respect to the nodal vector $\vec{\Psi}^T$, leads to the matrix equation

$$[\mathbf{K} - k_c^2 \mathbf{M}]\vec{\Psi} = 0, \quad \mathbf{K}_{JK} = \int\int_\Omega \nabla_t N_J \nabla_t N_K d\Omega,$$

$$\mathbf{M}_{JK} = \int\int_\Omega N_J N_K d\Omega, \vec{\Psi} = \begin{pmatrix} \Psi_1 \\ \Psi_1 \\ \vdots \\ \Psi_N \end{pmatrix}. \tag{4}$$

The generalized matrix eigenvalue problem (4) is reduced to tridiagonal form by the Lanczos procedure [9] with application of a shift and invert technique to accelerate convergence. Full Gram-Schmidt-type reorthogonalization guarantees the orthogonality of even higher-order multiple degenerate modes. The system of equations arising in each Lanczos iteration step is solved by sparse matrix Cholesky decomposition using the minimum degree algorithm [10].

Matching the transverse electro-magnetic fields $\vec{E}_t$ and $\vec{H}_t$ at the common interface of a general waveguide step discontinuity (see Fig. 1) leads to the set of matrix equations

$$\begin{aligned} \mathbf{D}_I(a_I + b_I) &= \mathbf{C}\mathbf{D}_{II}(a_{II} + b_{II}) \\ \mathbf{C}^T \mathbf{D}_I^{-1}(a_I - b_I) &= \mathbf{D}_{II}^{-1}(-a_{II} + b_{II}) \end{aligned} \tag{5}$$

with

$$\mathbf{C}_{mn} = \begin{cases} \int\int_{\Omega_{II}} \nabla_t \Psi_m^{HI} \nabla_t \Psi_n^{HII} d\Omega & \text{TE-TE} \\ \int\int_{\Omega_{II}} \nabla_t \Psi_m^{HI} (\vec{u}_z \times \nabla_t \Psi_n^{EII}) d\Omega (= 0) & \text{TE-TM} \\ \int\int_{\Omega_{II}} (\vec{u}_z \times \nabla_t \Psi_m^{EI}) \nabla_t \Psi_n^{HII} d\Omega & \text{TM-TE} \\ \int\int_{\Omega_{II}} \nabla_t \Psi_m^{EI} \nabla_t \Psi_n^{EII} d\Omega & \text{TM-TM} \end{cases},$$

$$\mathbf{D}_{I,II} = \operatorname{diag}\begin{pmatrix} \sqrt{\dfrac{\omega\mu_{I,II}}{\beta_{I,II}^H}} \\ \sqrt{\dfrac{\beta_{I,II}^E}{\omega\epsilon_{I,II}}} \end{pmatrix} \begin{matrix} \text{TE} \\ \text{TM} \end{matrix} \tag{6}$$

for the amplitude vectors $a_{I,II}$ and $b_{I,II}$ of the incident and scattered waves in waveguide I and II, respectively. From this set of equations, the generalized scattering matrix (GSM) of the complete step discontinuity can be obtained. The GSM of the whole structure is calculated by combination of the GSM's of the step discontinuities with the GSM's of the homogeneous waveguide sections between them [2], [3]. In this way, rather complicated structures can also be analyzed, such as filters of more than 2-pole complexity.

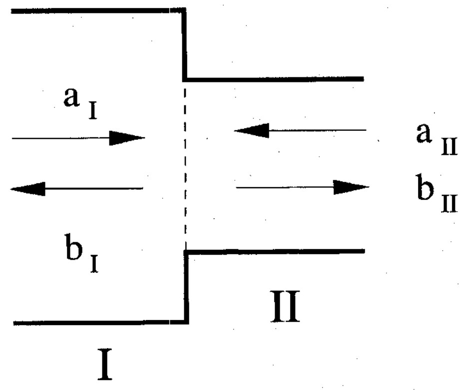

Fig. 1. Waveguide step discontinuity with incident and scattered waves.

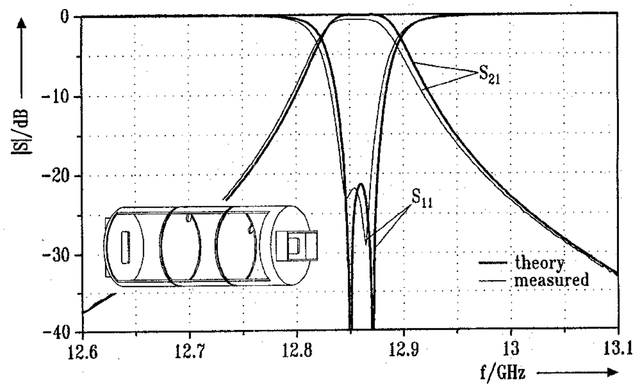

Fig. 2. Calculated and measured return and insertion losses of a circular waveguide dual-mode filter composed of an available cavity structure with three sections. Orthomode coupling provided, therefore, by two rectangular post elements with rounded edges, in a 30° and 60° position, respectively. Filter dimensions in mm: WR62 waveguide in- and output ports (15.799 × 7.899); irises: 9.5 × 5.5, radius 1.0, thickness $t = 0.15$; coupling posts: width 1.0, thickness $t = 0.15$, depth 2.165, radius of rounded edges 0.5; resonator lengths: first section 16.605, second section 17.219, third section 16.599, radius 6.985.

III. Results

For the verification of the theory, a dual-mode filter example has been chosen, Fig. 2, where the hardware was already available: a circular waveguide cavity consisting of three sections, rectangular WR62 waveguide (15.799 mm × 7.899 mm) in- and output ports twisted by 90° and rectangular coupling irises with rounded corners. For the coupling of the orthogonal modes, because of the already available triple cavity section, two rectangular post elements with rounded edges in a 30° (first element) and 60° position (second element), respectively, have been chosen (Fig. 2). These post elements have been fabricated by etching techniques. Excellent agreement between the calculated and measured insertion and return loss is demonstrated.

For the final analysis of the filter in Fig. 2, all higher-order modes have been considered with a cut-off frequency of approximately up to 15 f_0, where f_0 is the center frequency of the filter. The computation time per frequency point for the final analysis of the filter was then about 70 sec on a low-cost IBM-RS6000-58F workstation. The efficient conjugate gradient (CG) method [11] for the systems of equations (5) of the whole circular waveguide—ridged waveguide—circular

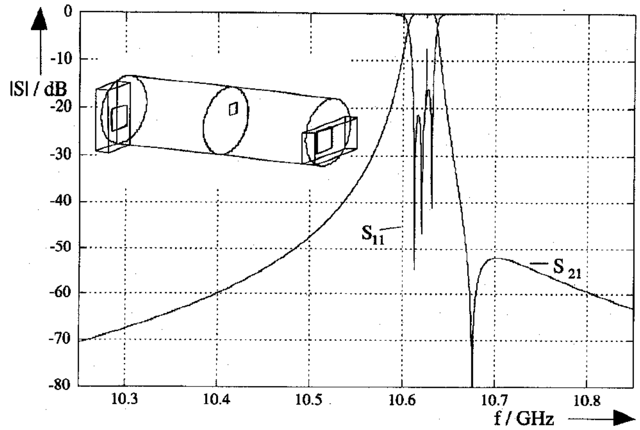

Fig. 3. Calculated return and insertion losses of a circular waveguide four-pole dual-mode filter with asymmetrically located irises. Filter dimensions in mm: WR75 waveguide in- and output ports (19.05 × 9.525); irises 1 and 3: 6.8 × 6.8, corner radius 0.5, thickness $t = 0.1$, located at $x = -2.0$, $y = -2.0$; iris 2: 3.6 × 3.6, corner radius 0.5, thickness $t = 0.1$, located at $x = 3.05$, $y = 3.05$; resonator length: both sections 24.5; radius: 10.

waveguide transition was applied. For the optimization of the filter, higher-order modes have been considered merely with a cut-off frequency of 10 f_0. Twenty-five frequency sample points and 100 iteration steps were necessary.

For also demonstrating the applicability of the presented technique for more complicated structures, Fig. 3 shows the design results of a not-yet-optimized four-pole filter where the dual-mode coupling is provided by asymmetrically located irises.

IV. Conclusion

An efficient hybrid mode-matching finite element method (MM/FEM) is applied for the rigorous modal analysis of complete circular waveguide dual-mode filters that can be coupled by irises with arbitrarily shaped cross-sections. The dual-mode coupling is provided either by asymmetrically located irises or, more traditionally, by obliquely positioned rectangular post elements with rounded edges that are an appropriate model for the usual coupling screws. The theory is verified by excellent agreement with measurements.

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